## 17. SYSTEM OF NONLINEAR EQUATIONS

Problem statement: Solution of equations: $f(x, y)=0$ $g(x, y)=0$
Symbolic solution: manipulation with expressions
Numeric solution: (i) separation of roots
(ii) iterative approximation of separate roots

### 17.1 Symbolic Solution

## Characteristics:

1. Symbolic solution is not always possible
2. Substitution allows conversion to numerical solution

## COMMANDS

## SYMS

SOLVE DOUBLE EZPLOT

FUNCTION
\% Example 17.1: Symbolic solution of system of nonlinear equations
$\%$ Equation1: $f(x, y)=x^{\wedge} 2-2 * x-y+0.5=0$
\% Equation2: $g(x, y)=x^{\wedge} 2+4 * y^{\wedge} 2-4=0$
syms x y
R1=solve (' $x^{\wedge} 2-2 * x-y+0.5=0$ ', ' $x^{\wedge} 2+4 * y^{\wedge} 2-4=0$ ');
R1.x; R1.y;
X1=double (R1.x), Y1=double(R1.y)
\% Visualization
ezplot (' $x^{\wedge} 2-2 * x-y+0.5=0$ ', $\left.\left[\begin{array}{ll}-1 & 3\end{array}\right]\right)$; grid on
hold on; ezplot (' $x^{\wedge} 2+4 * y^{\wedge} 2-4=0$ ') ;
plot (X1 ([1 4 4 ) , Y1 ([1 4 4 ) ,'or'); hold off
title('SYSTEM OF NONLINEAR EQUATIONS')

### 17.2 Numeric Solution

Principle of the Newton method for the system of equations:

1. Expansion of multivariable functions into Taylor series is used
2. Principle is the same as in the one dimensional case
$\% \% \%$ Example 17.2: Solution of system of nonlinear equations
$\% \% \%$ Equation1: $f(x, y)=x^{\wedge} 2-2 * x-y+0.5=0$
$\% \% \%$ Equation2: $g(x, y)=x^{\wedge} 2+4 * y^{\wedge} 2-4=0$
$\% \% \%$ for initial approximation $P$, accuracy eps and for
$\% \% \%$ maximum number of iterations $M$
$\mathrm{P}=\left[\begin{array}{ll}4 & 2\end{array}\right]^{\prime}$; eps=1e-12; $\mathrm{M}=40 ; \mathrm{PG}=\mathrm{P}$;
for $k=2: M$
$\mathrm{DF}=\mathrm{J}(\mathrm{P}) ; \mathrm{F}=[-\mathrm{f}(\mathrm{P}) ;-\mathrm{g}(\mathrm{P})]$;
$\mathrm{DP}=\operatorname{inv}(\mathrm{DF}) * \mathrm{~F}$;
$\mathrm{P}=\mathrm{P}+\mathrm{DP} ; \mathrm{PG}=[\mathrm{PG} \mathrm{P}]$;
if abs(sum(DP))<eps, break, end
end
$x k=P(:, e n d) ;$
plot(PG'); grid on; title('SOLUTION EVOLUTION')
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function $z=f(P)$ function $z=g(P)$
$\mathrm{x}=\mathrm{P}(1) ; \mathrm{y}=\mathrm{P}(2)$; $\mathrm{x}=\mathrm{P}(1) ; \mathrm{y}=\mathrm{P}(2)$;
$\mathrm{z}=\mathrm{x}^{\wedge} 2-2 * \mathrm{x}-\mathrm{y}+0.5 ; \quad \mathrm{z}=\mathrm{x}^{\wedge} 2+4 * \mathrm{y}^{\wedge} 2-4$;
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function $W=J(P)$

$\mathrm{x}=\mathrm{P}(1)$; $\mathrm{y}=\mathrm{P}(2)$;
$\mathrm{W}=[(2 * \mathrm{x}-2)(-1)$;
( $2 * \mathrm{x}$ ) ( $8 * \mathrm{y})]$;

## EXAMPLES 17

17.1 Evaluate symbolic solution of a selected system of nonlinear equations
17.2 Evaluate numeric solution of a selected system of nonlinear equations

